

NPTEL ONLINE CERTIFICATION COURSES

Management of Fixed Income Securities

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Lecture 36: Bonds with Embedded Options

CONCEPTS COVERED

- Call risk
- Valuation of callable bonds

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KEYWORDS

- Option adjusted spread
- Price compression
- Binomial interest rate tree

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Embedded Options

- Many bonds have embedded options features like call feature, put feature, sinking fund provisions etc.
- Call feature gives issuer the right to buy back the bond from the bond holder
- Put features gives the bond holder the right to sell the bond back to the issuer
- The inclusion of these features increases the call risk, put risk etc. and make the evaluation of bond more difficult

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Call Risk

- **Call Risk** is the uncertainty that the realized return will deviate from the expected return because the issuer calls the bond, forcing the investor to reinvest in a market with lower rates

- Some callable bonds can be called at any time, whereas for others the call is deferred for a certain period, giving the investor protection during the deferment period

- Some bonds, as part of their sinking fund arrangements, are retired over the life of the bond, usually with the issuer having the choice of purchasing the bonds directly at market prices or calling the bonds at a specified call price

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Call Risk

- When a bond is called, the investor's realized rate of return is affected in two ways.
- First, since the call price is typically above the bond's face value, the actual rate of return the investor earns for the period from the purchase of the bond to its call generally is greater than the yield on the bond at the time it was purchased.
- However, if an investor originally bought the bond because its maturity matched investor’s horizon date, then the investor will be facing with the disadvantage of reinvesting the call and investment proceeds at lower market rates.
- Moreover, this second effect, known as ***reinvestment risk***, often dominates the first effect, resulting in a rate of return over the investor's horizon that is lower than the promised YTM when the bond was bought.

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Example

Consider the case of an investor with a 10-year horizon who purchases a 10-year, 10% coupon bond at its par value of Rs. 1000, with coupon payments paid annually and with the bond callable at a call price (CP) of Rs. 1100. In addition, suppose that the yield curve for such bonds is flat at 10% and that it remains that way for the first three years the investor holds the bond. At the end of year three, however, assume the yield curve shifts down to 8% and the issuer calls the bond. What will be the investor's total return (TR) for the three-year period?

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Example Cont…

At the end of year three the investor's cash value would be:

1	2	3 (Call Date)
100		$100(1.1)^2 = 121$
	100	$100(1.1) = 110$
		$100+1100=1200$
	Total	1431

$$TR_3 = \left[\frac{1431}{1000}\right]^{1/3} - 1$$

$$TR_3 = 0.1269 = 12.69\%$$

So, for the call period, the investor earns a rate of return greater than the initial YTM (10%).

However, with a 10-year horizon, the investor must reinvest the Rs. 1431 cash for seven more years at the lower market rate.

If we assume she reinvests all coupons to the horizon date at 8%, then the Rs. 1431 will grow at an 8% annual rate to equal Rs. 2452.48 (= $Rs.1431(1.08)^7$) at the end of the tenth year, yielding a total return for the 10-year period of 9.386%:

$$TR_{10} = \left[\frac{1431(1.08)^7}{1000}\right] = \left[\frac{2452.48}{1000}\right]^{1/10} - 1 = 0.09386$$

$$TR_{10} = \left[(1.1269)^3(1.08)^7\right]^{1/10} - 1 = 0.09386$$

Call Risk

- With the total return of 9.386% less than the initial YTM of 10%, the second effect of reinvesting in a market with lower rates dominates the first effect of a call price greater than the bond's face value.

- This example shows that bonds that are callable are subject to the uncertainty that the actual rate will be less than the investor's expected YTM.

- Because of this call risk, there is usually a lower market demand and price for callable bonds than non-callable bonds, resulting in a higher rate of return or interest premium on callable over non-callable bonds.

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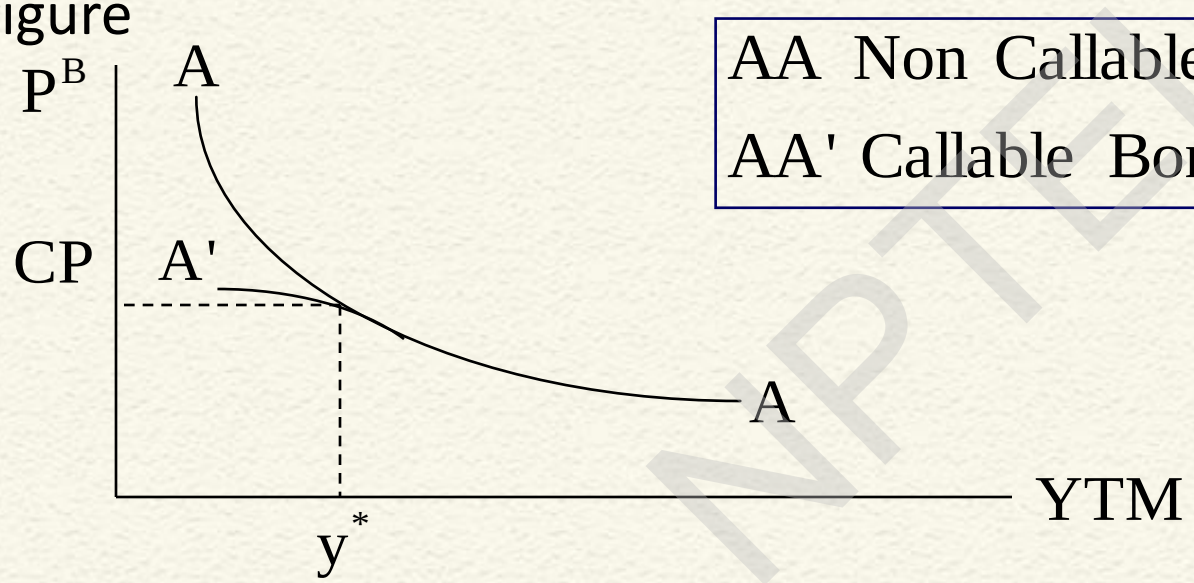
Call Risk

- The size of this interest premium, in turn, depends on investors' and borrowers' expectations concerning interest rates.
- When interest rates are high and expected to fall, bonds are more likely to be called; thus, in a period of high interest rates, a relatively low demand and higher rate on callable over noncallable bonds would occur.
- In contrast, when interest rates are low and expected to rise, we expect the effect of call provisions on interest rates to be negligible.
- The difference in yields between a bond with option features, such as a call option, sinking fund call, or put option, and an otherwise identical option-free bond is defined as the ***option adjusted spread*** (OAS).

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Price Compression

- Call features put limitations on the price-yield curve. At the rate where the bond could be called, the price-yield curve flattens, with the price equal to the CP. This limitation on the price is referred to as ***price compression.*** This limitation is illustrated in the figure



AA Non Callable Bond

AA' Callable Bond

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Price Compression

- In the figure, the price-yield curve AA is shown for a noncallable bond. This curve is negatively sloped and convex from below.

- The curve AA/ represents the price-yield curve for a comparable callable bond. As shown, this curve flattens out and becomes concave ***(negative convexity***) at rate y*, where y* represents a threshold rate that corresponds to a bond price equal, or approximately equal, to the call price.

- Since the callable bond would likely be called if rates are at y* or less, we would not expect investors to pay a price for such bond greater than the call price. Thus, the price-yield curve for the callable bond would tend to flatten out at y*.

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Valuation of Callable Bonds

- When valuing a callable bond, one needs to take into account the possibility that interest rates could decrease, leading to the bond being called. If called, the bond's cash flow patterns would be different than if rates increased and the bond was not called.

- Given the uncertainty of the bond's cash flows, valuing callable bonds and other bonds with embedded option features is more difficult than valuing option-free bonds.

- One approach to valuing callable bonds is to incorporate interest rate volatility by using a ***binomial interest rate tree***.

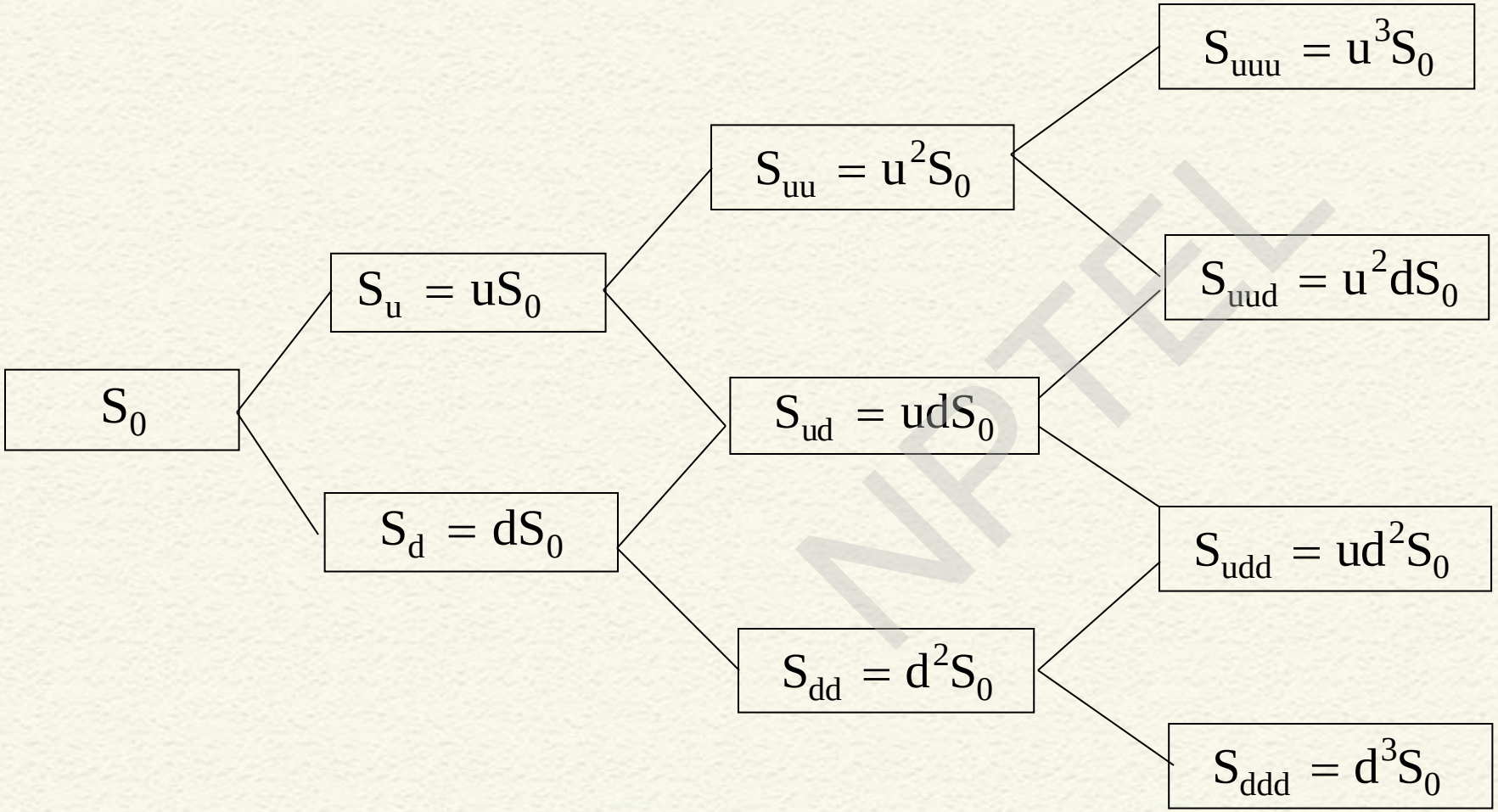
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Binomial Interest Rate Model

- A binomial model of interest rates assumes a spot rate of a given maturity follows a binomial process where in each period it has either a higher or lower rate.
- Assume a one-period, risk-free spot rate (S) follows a process in which in each period the rate is equal to a proportion u times its beginning-of-the-period value or a proportion d times its initial value, where u is greater than d.
- After one period, there would be two possible one-period spot rates: $S_u = uS_0$ and $S_d = dS_0$.
- If the proportions u and d are constant over different periods, then after two periods there would be three possible rates: $S_{uu} = u^2S_0$, $S_{ud} = udS_0$, or $S_{dd} = d^2S_0$.

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Binomial Interest Rate Model



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Example

Given

The current one-period spot rate: $S_0 = 10\%$

The upward parameter: $u = 1.1$

The downward parameter: $d = 0.95$

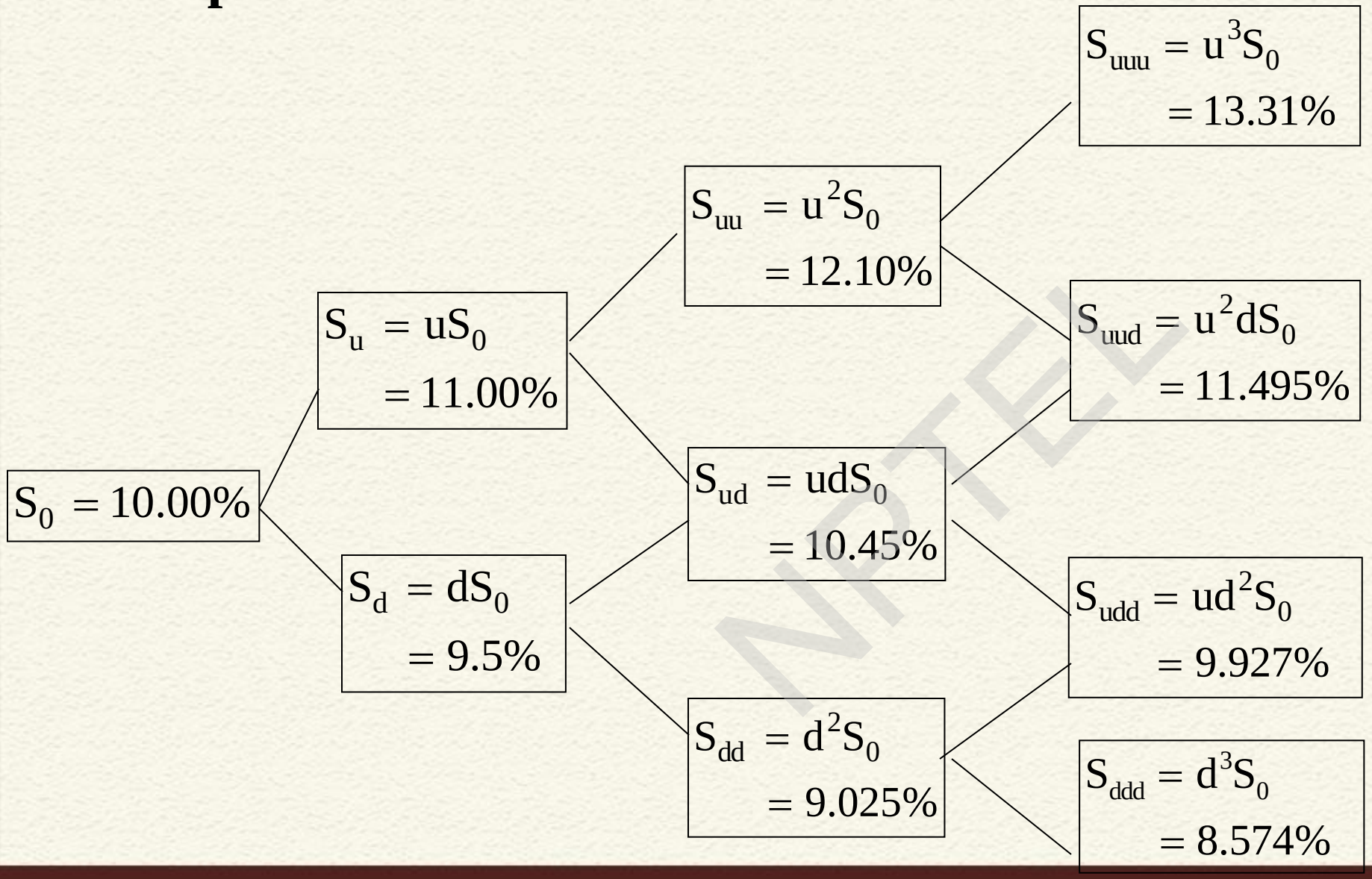
What are the two possible one-period rates after one period?

What are the three possible one-period rates after two period?

What are the four possible one-period rates after three periods?

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Example Cont…



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CONCLUSIONS

- Call Risk is the uncertainty that the realized return will deviate from the expected return because the issuer calls the bond, forcing the investor to reinvest in a market with lower rates
- Call features put limitations on the price-yield curve and this limitation on the price is referred to as price compression
- Binomial interest rate tree approach is used for valuation of callable bonds

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REFERENCES

- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Fabozzi, J. Frank and Mann, V. Steven (2005): The Hand Book of Fixed Income Securities, Tata McGraw-Hill, 7th Edition.

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Lecture 37: Binomial Valuation of Callable Bonds-I

CONCEPTS COVERED

- Valuation of two period option free bonds
- Valuation of two period callable bonds

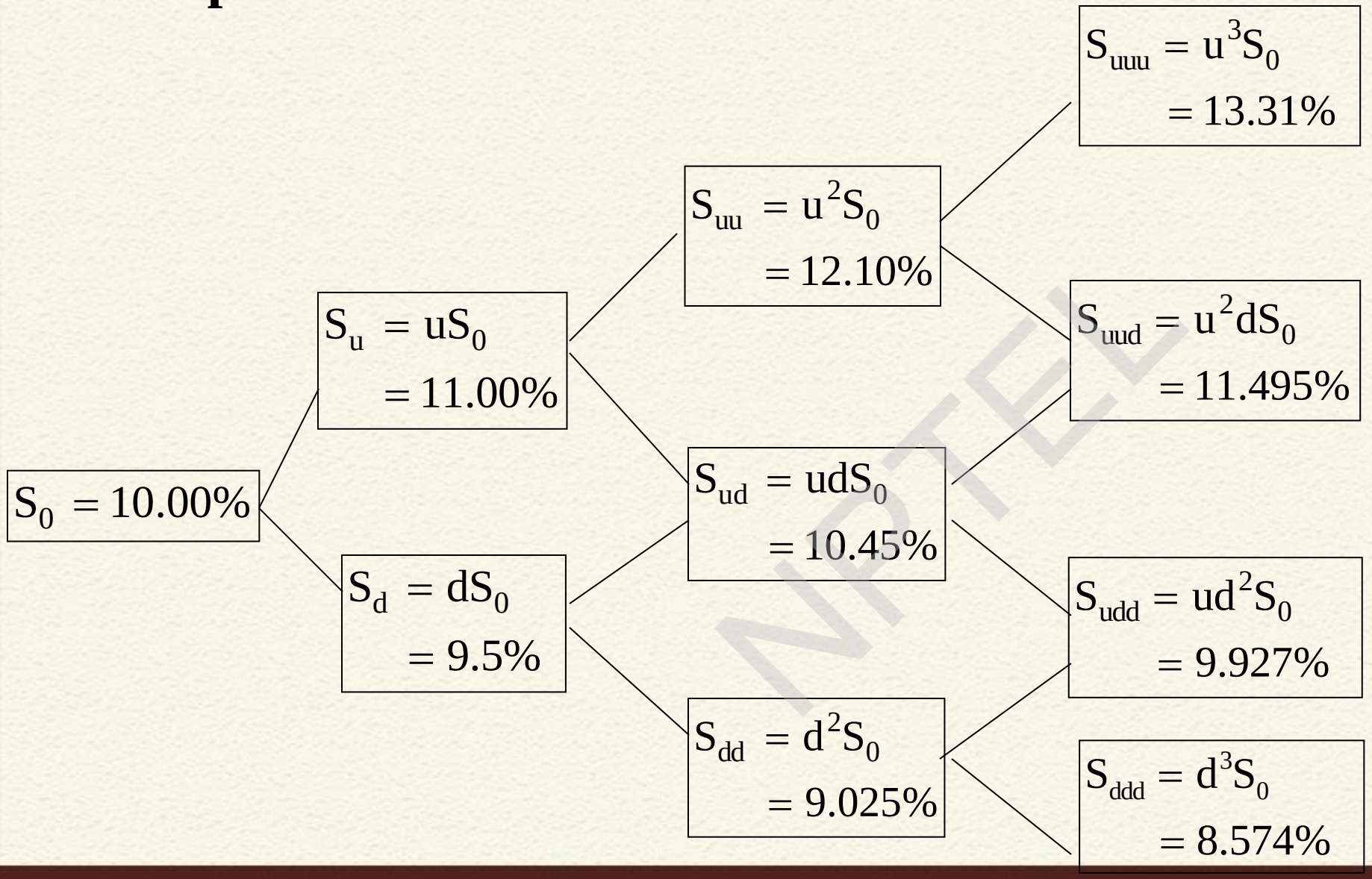
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KEYWORDS

- Call option
- Call price
- Spot rates
- Intrinsic value

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Example Cont…



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Valuation of Two-Period Option-Free Bond Example

Given the possible one-period spot rates, suppose we want to value a bond with the following features:

- The bond matures in two periods
- The bond has no default risk
- The bond has no embedded option features – option-free bond
- The bond pays an 8% coupon each period
- The bond pays a Rs 100 principal at maturity

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Valuation of Two-Period Option-Free Bond Example

- Since there is no default or call risk, the only risk an investor assumes in buying this bond is market risk. This risk occurs at time period one.
- At that time, the original two-period bond will have one period to maturity where there is a certain payoff of 108. We don't know, though, whether the one-period rate will be 11% or 9.5%:
- If the ***rate is 11%,*** then the bond would be worth 97.297:

$$B_u = 108/1.11 = 97.297$$

- If the ***rate is 9.5%,*** the bond would be worth 98.630:

$$B_d = 108/1.095 = 98.630$$

Valuation of Two-Period Option-Free Bond Example

- Given the two possible values in Period 1, the current value of the two-period bond can be found by calculating the ***present value of the bond's expected cash flows*** in Period 1.

- If we assume that there is an equal probability (pr) of the one-period spot rate being higher ***(pr = 0.5)*** or lower ***(1 – pr = 0.5),*** then the current value of the two-period bond (B_0) would be:

$$B_0 = \frac{pr[B_u + C] + (1 - pr)[B_d + C]}{1 + S_0}$$

$$B_0 = \frac{0.5[97.297 + 8] + 0.5[98.630 + 8]}{1.10} = 96.330$$

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Valuation of Two-Period Option-Free Bond Example

$Coupon = 8,\ F = 100$

$S_0 = 0.10, u = 1.1,\ d = 0.95$

Valuing a two-Period Option-Free Bond

$S_0 = 10\%$

$B_0 = \dfrac{0.5(97.297+8) + 0.5(98.630+8)}{(1.10)}$

$= 96.330$

$S_u = 11\%$

$B_u = \dfrac{108}{(1.11)} = 97.297$

$S_d = 9.5\%$

$B_d = \dfrac{108}{(1.095)} = 98.630$

$B_{uu} = 108$

$B_{ud} = 108$

$B_{dd} = 108$

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Valuation of Two-Period Callable Bond

- Suppose that the two-period, 8% bond has a call feature that allows the issuer to buy back the bond at a call price (CP) of 98.

- Using the binomial tree approach, this call option can be incorporated into the valuation of the bond by determining at each node in Period 1 whether or not the issuer would exercise his right to call.

- The issuer will find it profitable to exercise whenever the bond price is above the call price (assuming no transaction or holding costs).

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Valuation of Two-Period Callable Bond

- In general, since the bond is only exercised when the call price is less than the bond value, the value of the callable bond in Period 1 is therefore the ***minimum*** of its call price or its binomial value:

$$B_t^C = \text{Min}[B_t, CP]$$

Rolling the two callable bond values in Period 1 of 97.297 and 98 to the present, we obtain a current price of 96.044:

$$B_0^C = \frac{0.5[97.297 + 8] + 0.5[98 + 8]}{1.10} = 96.044$$

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Valuation of Two-Period Callable Bond Example

$$\text{Coupon} = 8,\ F = 100,\ CP = 98$$
$$S_0 = .10,\ u = 1.1,\ d = .95$$

Valuing a Two-Period Callable Bond

$$S_0 = 10\%$$
$$B_0 = \frac{0.5(97.297+8) + 0.5(98+8)}{(1.10)}$$
$$= 96.044$$

$$S_u = 11\%$$
$$B_u = \frac{108}{(1.11)} = 97.297$$
$$B_u^C = \text{Min}[B_u, CP]$$
$$B_u^C = \text{Min}[97.297, 98] = 97.297$$

$$S_d = 9.5\%$$
$$B_d = \frac{108}{(1.095)} = 98.630$$
$$B_d^C = \text{Min}[B_d, CP]$$
$$B_d^C = \text{Min}[98.630, 98] = 98$$

$$B_{uu} = 108$$

$$B_{ud} = 108$$

$$B_{dd} = 108$$

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Valuation of Two-Period Callable Bond Example

- The bond's embedded call option lowers the value of the bond from 96.330 to 96.044.

- At each of the nodes in Period 1, the value of the callable bond is determined by selecting the minimum of the binomial bond value or the call price, and then rolling the callable bond value to the current period.

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Value of Two-Period Callable Bond

- When the one-period spot rate is 9.5% in Period 1 and the bond is priced at 98.630, it is profitable for the issuer to call the bond. The price of the bond in this case would be the call price of 98.
- When the one-period spot rate is 11% in Period 1, it is not profitable for the issuer to exercise the call. The price of the bond in this case remains at 97.297.
- In the 9.5% case, the issuer could buy the bond back at 98 financed by issuing a one-year bond at 9.5% interest.
- One period later the issuer would owe 98(1.095) = 107.31
- This represents a ***savings*** of 108 − 107.31 = 0.69
- The present value of that savings in Period 1 is 0.69/1.095 = 0.63
- 0.63 is equal to the difference between the bond price and the call price: 98.630 − 98 = 0.63.

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Alternative Approach: Valuation of Two-Period Callable Bond

- Instead of using a price constraint at each node, the price of the callable bond can alternatively be found by determining the ***value of the call option*** at each node, V_t^C, and then subtracting that value from the noncallable bond value:

$$B_t^C = B_t^{NC} - V_t^C$$

- In this two-period case, the values of the call option are equal to their intrinsic values (IV) (or exercise values). The intrinsic value is the maximum of B_t^{NC} - CP or zero:

$$V_t^C = \text{Max}[B_t^{NC} - CP, 0]$$

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Alternative Approach: Valuation of Two-Period Callable Bond Example

- The two possible call values in Period 1 are 0 and 0.63, and
- The two corresponding callable bond values are 97.297 and 98.
- The value of the call option in the current period is equal to the present value of the expected call value in Period 1

$$V_0^C = \frac{0.5[0] + 0.5[.630]}{1.10} = 0.2864$$

- Subtracting the call value of .2864 from the noncallable bond value of 96.330, we obtain a callable bond value

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Alternative Approach: Valuation of Two-Period Callable Bond Example

$$S_0 = 10\%,\ B_0^{NC} = 96.330$$

$$V_0^C = \frac{0.5(0)+0.5(0.630)}{(1.10)} = 0.2864$$

$$B_0^C = B_0^{NC} - V_0^C$$

$$= 96.330 - 0.2864 = 96.044$$

$$S_u = 11\%,\ B_u^{NC} = 97.297$$

$$V_u^C = \text{Max}[B_u - CP, 0]$$

$$V_u^C = \text{Max}[97.297 - 98, 0] = 0$$

$$B_u^C = B_u^{NC} - V_u^C$$

$$= 97.297 - 0 = 97.297$$

$$S_d = 9.5\%,\ B_d^{NC} = 98.630$$

$$V_d^C = Max[B_d - CP, 0]$$

$$V_d^C = Max[98.630 - 98, 0] = 0.630$$

$$B_d^C = B_d^{NC} - V_d^C$$

$$= 98.630 - 0.630 = 98$$

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CONCLUSIONS

- The issuer finds it profitable to exercise the call option whenever the bond price is above the call price
- As the bond is only exercised when the call price is less than the bond value, the value of the callable bond in Period 1 is the minimum of its call price or its binomial value
- The value of call option is the difference between the value of noncallable bond and value of call option at each node

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REFERENCES

- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
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Lecture 38: Binomial Valuation of Callable Bonds-II

CONCEPTS COVERED

- Valuation of three period option free bonds
- Valuation of three period callable bonds

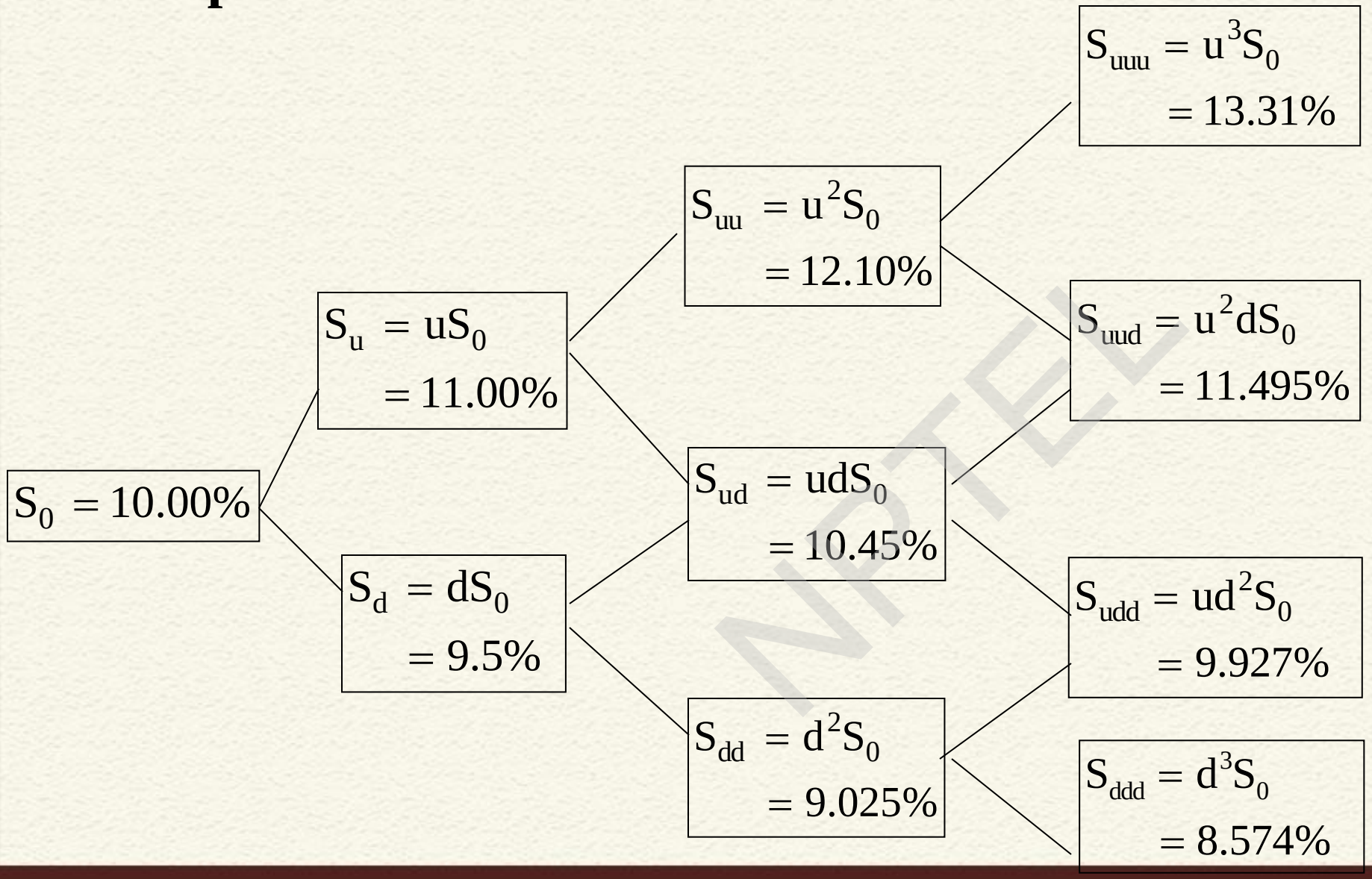
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KEYWORDS

- Exercise value
- Upper node
- Lower node

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Example Cont…



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Valuation of a Three-Period Option Free Bond

- If we want to value a three-period bond, we need a two-period interest rate tree.
- Suppose we want to value a three-period, 9% coupon bond with no default risk or option features.
- In this case, market risk exists in two periods: Period 3, where there are three possible spot rates, and Period 2, where there are two possible rates.

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Valuation of a Three-Period Option Free Bond

1. We first determine the three possible values of the bond in Period 2 given the three possible spot rates and the bond's certain cash flow next period. The three possible values in Period 2 are:

$$B_{uu} = 109/1.121 = 97.2346$$
$$B_{ud} = 109/1.1045 = 98.6872$$
$$B_{dd} = 109/1.09025 = 99.9771$$

2. Given these values, we next roll the tree to the first period and determine the two possible values there as the present values of the expected cash flows in Period 2:

$$B_u = \frac{0.5[97.2346+9] + 0.5[98.6872+9]}{1.11} = 96.3612$$

$$B_d = \frac{0.5[98.6872+9] + 0.5[99.9771+9]}{1.095} = 98.9335$$

3. Finally, using the bond values in Period 1, we roll the tree to the current period where we determine the value of the bond

$$B_0 = \frac{0.5[96.3612+9] + 0.5[98.9335+9]}{1.10} = 96.9521$$

Valuation of a Three-Period Option Free Bond

$C = 9,\ F = 100$

$S_0 = 0.10,\ u = 1.1,\ d = 0.95$

$S_0 = 10\%$

$$B_0 = \frac{0.5(96.3612 + 9) + 0.5(98.9335 + 9)}{(1.10)} = 96.9521$$

$S_u = uS_0 = 11\%$

$$B_u = \frac{0.5(97.2346 + 9) + 0.5(98.6872 + 9)}{(1.11)} = 96.3612$$

$S_d = dS_0 = 9.5\%$

$$B_d = \frac{0.5(96.6872 + 9) + 0.5(99.9771 + 9)}{(1.095)} = 98.9335$$

$S_{uu} = u^2 S_0 = 12.1\%$

$$B_{uu} = \frac{109}{1.121} = 97.2346$$

$S_{ud} = udS_0 = 10.45\%$

$$B_{ud} = \frac{109}{1.1045} = 98.6872$$

$S_{dd} = d^2 S_0 = 9.025\%$

$$B_{dd} = \frac{109}{1.09025} = 99.9771$$

109

109

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Valuation of a Three-Period Callable Bond

- First compare each of the noncallable bond values with the call price in Period 2 (one period from maturity); then take the minimum of the two as the callable bond value.

- Next roll the callable bond values from Period 2 to Period 1 and determine the two bond values at each node as the present value of the expected cash flows, and then for each case select the minimum of the value calculated or the call price.

- Finally, roll those two callable bond values to the current period and determine the callable bond's price as the present value of Period 1's expected cash flows.

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Valuation of a Three-Period Callable Bond Example

- The binomial tree value of the three-period, 9% bond given a call feature with a CP = 98
- At the two lower nodes in Period 2, the bond would be called at 98 and therefore the callable bond price would be 98; at the top node in Period 2, the bond price of 97.2346 would prevail.
- Rolling Period 2's prices to Period 1, the present values of the expected cash flows are 96.0516 at the 11% spot rate and 97.7169 at the 9.5% rate. Since neither of these values are less than the CP of 98, each represents the callable bond value at that node.
- Rolling Period 1's two values to the current period, we obtain the value for the three-period callable bond.

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Valuation of a Three-Period Callable Bond Example

$C = 9,\ F = 100,\ CP = 98$

$S_0 = 0.10,\ u = 1.1,\ d = 0.95$

$S_0 = 10\%$

$B_0^C = \dfrac{0.5(96.0516+9)+0.5(97.7169+9)}{(1.10)}$

$= 96.258$

$S_u = uS_0 = 11\%$

$B_u = \dfrac{0.5(97.2346+9)+0.5(98.+9)}{(1.11)}$

$= 96.0516$

$B_u^C = Min[B_u, CP]$

$B_u^C = Min[96.0516, 98] = 96.0516$

$S_d = dS_0 = 9.5\%$

$B_d = \dfrac{0.5(98+9)+0.5(98+9)}{(1.095)}$

$= 97.7169$

$B_d^C = Min[B_d, CP]$

$B_d^C = Min[97.7169, 98] = 97.7169$

$S_{uu} = u^2 S_0 = 12.1\%$

$B_{uu} = \dfrac{109}{1.121} = 97.2346$

$B_{uu}^C = Min[B_{uu}, CP]$

$B_{uu}^C = Min[97.2346, 98]$

$= 97.2346$

$S_{ud} = udS_0 = 10.45\%$

$B_{ud} = \dfrac{109}{1.1045} = 98.6872$

$B_{ud}^C = Min[B_{ud}, CP]$

$B_{ud}^C = Min[98.6872, 98]$

$= 98$

$S_{dd} = d^2 S_0 = 9.025\%$

$B_{dd} = \dfrac{109}{1.09025} = 99.9771$

$B_{dd}^C = Min[B_{dd}, CP]$

$B_{dd}^C = Min[99.9771, 98]$

$= 98$

109

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Alternative Approach: Valuation of Three-Period Callable Bond

- The alternative approach to valuing the callable bond is to determine the value of the call option at each node and then subtract that value from the noncallable value to obtain the callable bond's price.
- The intrinsic value or exercising value:

$$IV = Max[B_t^{NC} - CP, 0]$$

- Different from our previous two-period case, when there are three periods or more, we need to take into account that prior to maturity the bond issuer has two choices: The investor can either exercise the option or the investor can hold it for another period
- The value of holding, V_H, is the present value of the expected call value next period:

$$V_H = \frac{\Pr V_u^C + (1 - pr)V_d^C}{1 + S}$$

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Alternative Approach: Valuation of Three-Period Callable Bond

1. In Period 2 the value of holding is zero at all three nodes since next period is maturity where it is too late to call. The issuer, though, would find it profitable to exercise in two of the three cases where the call price is lower than the bond values. The three possible callable bond values in Period 2 are:

$$B_{uu}^{C} = B_{uu}^{NC} - V_{uu}^{C} = 97.2346 - \text{Max}[97.2346 - 98, 0] = 97.2346$$

$$B_{ud}^{C} = B_{ud}^{NC} - V_{ud}^{C} = 98.6872 - \text{Max}[98.6872 - 98, 0] = 98$$

$$B_{dd}^{C} = B_{dd}^{NC} - V_{dd}^{C} = 99.9771 - \text{Max}[99.9771 - 98, 0] = 98$$

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Alternative Approach: Valuation of Three-Period Callable Bond

2. In Period 1 at the lower node:
   The noncallable bond price is greater than the call price.
   In this case, the IV is 98.9335 − 98 = 0.9335.
   The value of holding the call, though, is 1.2166:

$$V_H = \frac{0.5[Max[98.6872-98,0]]+0.5[Max[99.9771-98,0]]}{1.095} = 1.2166$$

Thus, the issuer would find it more valuable to defer the exercise one period. As a result, the value of the call option is 1.2166 and the value of the callable bond is 97.7169

$$\text{Max}[\text{IV}, \text{V}_\text{H}] = \text{Max}[.8394, 1.2166] = 1.2166$$

$$\text{B}_\text{d}^\text{C} = \text{B}_\text{d}^\text{NC} - \text{V}_\text{d}^\text{C} = 98.9335 - 1.2166 = 97.7169$$

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Alternative Approach: Valuation of Three-Period Callable Bond

3. In Period 1 at the upper node

The price of the noncallable is 96.3612

The exercise value is 0.

The value of the call option in this case is equal to its holding value of 0.3095:

$$V_H = \frac{0.5[Max[97.2346 - 98,0]] + 0.5[Max[98.6872 - 98,0]]}{1.11} = 0.3095$$

The value of the callable bond is :

$$B_u^C = B_u^{NC} - V_u^C = 96.3612 - 0.3095 = 96.0517$$

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Alternative Approach: Valuation of Three-Period Callable Bond

4. Current Period:

Rolling the two possible option values of 0.3095 and 1.2166 in Period 1 to the current period, we obtain the current value of the option and the same callable bond value using the first approach:

$$V_0^C = \frac{0.5[.3095] + 0.5[1.2166]}{1.10} = 0.6937$$

$$B_0^C = B_0^{NC} - V_0^C = 96.9521 - 0.6937 = 96.2584$$

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Alternative Approach: Valuation of Three-Period Callable Bond

$C = 9,\ F = 100,\ CP = 98$

$S_0 = 0.10,\ u = 1.1,\ d = 0.95$

$S_0 = 10\%,\ B_0^{NC} = 96.9521$

$V_0^C = V_H = \dfrac{0.5(.3095) + 0.5(1.2166)}{(1.10)} = 0.6937$

$B_0^C = B_0^{NC} - V_0^C$

$= 96.9521 - 0.6937$

$= 96.2584$

$S_u = 11\%,\ B_u^{NC} = 96.3612$

$V_H = \dfrac{0.5(0) + 0.5(.6872)}{(1.11)} = 0.3095$

$IV = Max[B_u^{NC} - CP,\ 0]$

$= Max[96.3612 - 98, 0] = 0$

$V_u^C = Max[V_H, IV] = Max[.3095, 0] = 0.3095$

$B_u^C = B_u^{NC} - V_u^C = 96.3612 - 0.3095 = 96.0516$

$S_d = 9.5\%,\ B_d^{NC} = 98.9335$

$V_H = \dfrac{0.5(.6872) + 0.5(1.9771)}{(1.095)} = 1.2166$

$IV = Max[B_d^{NC} - CP,\ 0]$

$= Max[98.9335 - 98, 0] = 0.9335$

$V_d^C = Max[V_H, IV] = Max[1.2166, .9335] = 1.2166$

$B_d^C = B_d^{NC} - V_d^C = 98.9335 - 1.2166 = 97.7169$

$S_{uu} = 12.1\%,\ B_{uu}^{NC} = 97.2346$

$IV = \text{Max}[B_{uu}^{NC} - CP, 0]$

$= \text{Max}[97.2346 - 98, 0] = 0$

$V_{uu}^C = IV = 0$

$B_{uu}^C = B_{uu}^{NC} - V_{uu}^C$

$= 97.2346 - 0 = 97.2346$

$S_{ud} = 10.45\%,\ B_{ud}^{NC} = 98.6872$

$IV = Max[B_{ud}^{NC} - CP, 0]$

$= Max[98.6872 - 98, 0] = 0.6872$

$V_{ud}^C = IV = 0.6872$

$B_{ud}^C = B_{ud}^{NC} - V_{ud}^C$

$= 98.6872 - 0.6872 = 98$

$S_{dd} = 9.025\%,\ B_{dd}^{NC} = 99.9771$

$IV = \text{Max}[B_{dd}^{NC} - CP, 0]$

$= \text{Max}[99.9771 - 98, 0] = 1.9771$

$V_{dd}^C = IV = 1.9771$

$B_{dd}^C = B_{dd}^{NC} - V_{dd}^C$

$= 99.9771 - 1.9771 = 98$

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CONCLUSIONS

- Following the binomial tree model, the value of the three period callable bonds can be determined by discounting the present value of expected cash flows in each node in each period
- For each case the minimum of the value calculation or the call price is selected
- Alternative approach for the valuation of callable bond is to determine the call option at each node and subtract the value from the noncallable value

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- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
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Management of Fixed Income Securities

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Lecture 39: Binomial Valuation of Puttable and Sinking-Fund Bonds

CONCEPTS COVERED

- Binomial valuation of putable bond
- Binomial Valuation of Sinking-Fund Bond

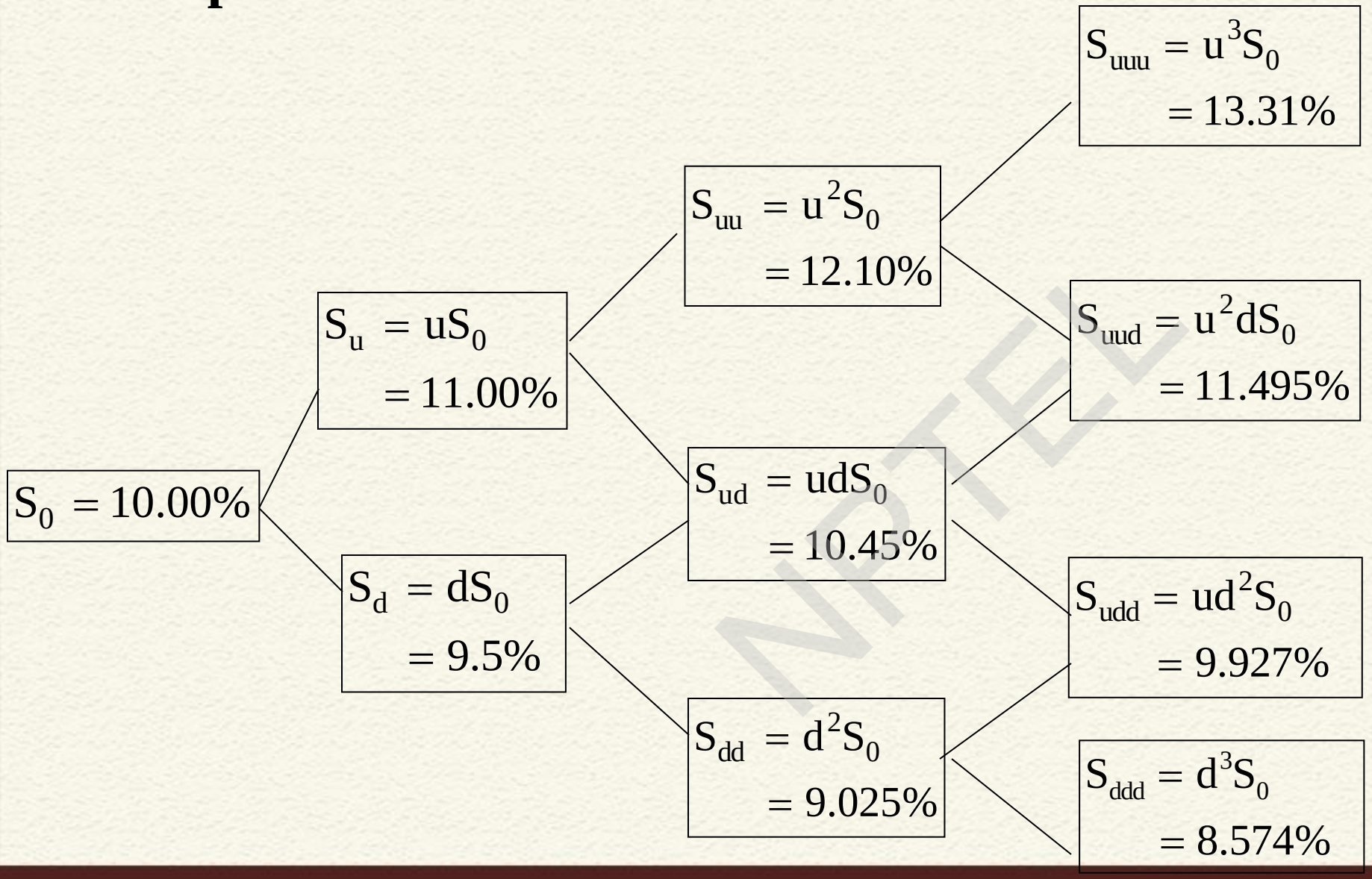
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KEYWORDS

- **Put option**
- **Sinking fund**

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Example Cont…



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Putable Bond

- A putable bond, or put bond, gives the holder the right to sell the bond back to the issuer at a specified exercise price (or put price)
- In contrast to callable bonds, putable bonds benefit the holder
- If the price of the bond decreases below the exercise price, then the bondholder can sell the bond back to the issuer at the exercise price
- From the bondholder's perspective, a put option provides a hedge against a decrease in the bond price.
- If rates decrease in the market, then the bondholder benefits from the resulting higher bond prices.
- If rates increase, then the bondholder can exercise, giving her downside protection.

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Binomial Valuation of Putable Bond

- Given that the bondholder has the right to exercise, the price of a putable bond will be equal to the price of an otherwise identical nonputable bond plus the value of the put option (V_0^P)

$$B_0^P = B_0^{NP} + V_0^P$$

- Since the bondholder will find it profitable to exercise whenever the put price exceeds the bond price, the value of a putable bond can be found using the binomial approach by comparing bond prices at each node with the put price and selecting the ***maximum*** of the two:

$$B_t^P = \text{Max}[B_t, PP]$$

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Binomial Valuation of Putable Bond

- The same binomial value can also be found by determining the value of the put option at each node and then pricing the putable bond as the value of an otherwise identical nonputable bond plus the value of the put option.

- In using the second approach, the value of the put option will be the maximum of either its intrinsic value (or exercising value), IV = Max[PP − B_t,0], or its holding value (the present value of the expected put value next period).

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Binomial Valuation of Three Period Putable Bond Example

- Suppose the three-period, 9% option-free bond in our previous example had a put option giving the bondholder the right to sell the bond back to the issuer at an exercise price of PP = 97 in Periods 1 or 2.

Period-2

- Using the two-period tree of one-period spot rates and the corresponding bond values for the option-free bond we start at Period 2 and investigate each of the nodes to determine if there is an advantage for the holder to exercise.
- In all three of the cases in Period 2, the bond price exceeds the exercise price; thus, there are no possible exercise advantages in this period and each of the possible prices of the putable bond are equal to their nonputable values and the values of each of the put options are zero.

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Binomial Valuation of Three Period Putable Bond Example

Period 1: Upper Node

- In Period 1 it is profitable for the holder to exercise when the spot rate is 11%. At that node, the value of the nonputable bond is 96.3612, compared to PP = 97; thus the value of putable bond is its exercise price of 97:

$$B_u^P = \text{Max}[96.3612, 97] = 97$$

- The putable bond price of 97 can also be found by subtracting the value of the put option from the price of the non-putable bond. The value of the put option at this node is 0.6388; thus, the value of the putable bond is 97:

$$V_u^P = Max[IV, V_H] = Max[97 - 96.3612,\ 0] = 0.6388$$

$$B_u^P = B_u^{NP} + V_u^P$$

$$B_u^P = 96.3612 + 0.6388 = 97$$

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Binomial Valuation of Three Period Putable Bond Example

Period-1: Lower Node

- At the lower node in Period 1, it is not profitable to exercise nor is there any holding value of the put option since there is no exercise advantage in Period 2. Thus at the lower node, the non-putable bond price prevails.

Current Period

- Rolling the two putable bond values in Period 1 to the present, we obtain a current value of the putable bond

$$B_0^P = \frac{0.5[97+9] + 0.5[98.9335+9]}{1.10} = 97.2425$$

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Binomial Valuation of Three Period Putable Bond Example (Alternative Approach)

- The current value also can be obtained using the alternative approach by computing the present value of the expected put option value in Period 1 and then adding that to the current value of the nonputable bond. With possible exercise values of 0.6388 and 0 in Period 1, the current put option value is:

$$V_0^P = \frac{0.5[0.6388] + 0.5[0]}{1.10} = 0.2904$$

$$B_0^P = B_0^{NP} + V_0^P = 96.9521 + 0.2904 = 97.2425$$

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Binomial Valuation of Three Period Putable Bond Example

$C = 9,\ F = 100,\ PP = 97$

$S_0 = 0.10,\ u = 1.1,\ d = 0.95$

$S_0 = 10\%$

$B_0^P = \dfrac{0.5(97+9)+0.5(98.9335+9)}{(1.10)}$

$= 97.2425$

$S_u = uS_0 = 11\%$

$B_u = \dfrac{0.5(97.2346+9)+0.5(98.6872+9)}{(1.11)}$

$= 96.3612$

$B_u^P = Max[B_u, PP]$

$B_u^P = Max[96.3612, 97] = 97$

$S_d = dS_0 = 9.5\%$

$B_d = \dfrac{0.5(98.6872+9)+0.5(99.9771+9)}{(1.095)}$

$= 98.9335$

$B_d^P = Max[B_d, PP]$

$B_d^P = Min[98.9334, 97] = 98.9335$

$S_{uu} = u^2 S_0 = 12.1\%$

$B_{uu} = \dfrac{109}{1.121} = 97.2346$

$B_{uu}^P = Max[B_{uu}, PP]$

$B_{uu}^P = Max[97.2346, 97]$

$= 97.2346$

$S_{ud} = udS_0 = 10.45\%$

$B_{ud} = \dfrac{109}{1.1045} = 98.6872$

$B_{ud}^P = Max[B_{ud}, PP]$

$B_{ud}^P = Max[98.6872, 97]$

$= 98.6872$

$S_{dd} = d^2 S_0 = 9.025\%$

$B_{dd} = \dfrac{109}{1.09025} = 99.9771$

$B_{dd}^P = Max[B_{dd}, PP]$

$B_{dd}^P = Max[99.9771, 97]$

$= 99.9771$

109

109

109

109

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Binomial Valuation of Three Period Putable Bond Example (Alternative Approach)

$S_0 = 10\%,\ B_0^{NP} = 96.9521$

$V_0^P = V_H = \dfrac{0.5(0.6388)+0.5(0)}{(1.10)} = 0.2904$

$B_0^P = B_0^{NP} + V_0^P$

$= 96.9521 + 0.2904$

$= 97.2425$

$S_u = 11\%,\ B_u^{NP} = 96.3612$

$V_H = \dfrac{0.5(0)+0.5(0)}{(1.11)} = 0$

$IV = Max[PP - B_u^{NP},\ 0]$

$= Max[97 - 96.3612, 0] = 0.6388$

$V_u^P = Max[V_H, IV] = Max[0, 0.6388] = 0.6388$

$B_u^P = B_u^{NP} + V_u^P = 96.3612 + 0.6388 = 97$

$S_d = 9.5\%,\ B_d^{NP} = 98.9335$

$V_H = \dfrac{0.5(0)+0.5(0)}{(1.095)} = 0$

$IV = Max[PP - B_d^{NP},\ 0]$

$= Max[97 - 98.9335, 0] = 0$

$V_d^P = Max[V_H, IV] = Max[0, 0] = 0$

$B_d^P = B_d^{NP} + V_d^P = 98.9335 + 0 = 98.9335$

$S_{uu} = 12.1\%,\ B_{uu}^{NP} = 97.2346$

$IV = \text{Max}[PP - B_{uu}^{NP}, 0]$

$= \text{Max}[97 - 97.2346, 0] = 0$

$V_{uu}^P = IV = 0$

$B_{uu}^P = B_{uu}^{NP} + V_{uu}^P$

$= 97.2346 + 0 = 97.2346$

$S_{ud} = 10.45\%,\ B_{ud}^{NP} = 98.6872$

$IV = \text{Max}[PP - B_{ud}^{NP}, 0]$

$= \text{Max}[97 - 98.6872, 0] = 0$

$V_{ud}^P = IV = 0$

$B_{ud}^P = B_{ud}^{NP} + V_u^P$

$= 98.6872 + 0 = 98.6872$

$S_{dd} = 9.025\%,\ B_{dd}^{NP} = 99.9771$

$IV = \text{Max}[PP - B_{dd}^{NP}, 0]$

$= \text{Max}[97 - 99.9771, 0] = 0$

$V_{dd}^P = IV = 0$

$B_{dd}^P = B_{dd}^{NP} + V_{dd}^P$

$= 99.9771 + 0 = 99.9771$

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Sinking-Fund Bonds

- Sinking fund arrangement in which the issuer has the option to buy some of the bonds back either at their market price or at a call price
- The issuer makes scheduled payments into a fund or buy up a certain proportion of the bond issue each period.
- Often when the sinking fund agreement specifies an orderly retirement of the issue, the issuer is given an option of either purchasing the bonds in the market or calling the bonds at a specified call price

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Sinking-Fund Bonds

- The call option embedded in the sinking fund agreement makes the sinking fund valuable to the issuer.
- If interest rates are relatively high, then the issuer will be able to buy back the requisite amount of bonds at a relatively low market price.
- If rates are low and the bond price high, though, then the issuer will be able to buy back the bonds on the call option at the call price.
- Thus, a sinking fund bond with this type of call provision should trade at a lower price than an otherwise identical non-sinking fund bond.

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Binomial Valuation of a Sinking-Fund Bond

- Since the sinking fund arrangement requires an immediate exercise or bond purchase at the specified sinking fund dates, the possible values of the sinking fund's call features at those dates are equal to their intrinsic values.

- This differs from the valuation of a standard callable bond where a holding value is also considered in determining the value of the call option.

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Binomial Valuation of a Sinking-Fund Bond (Example)

- Suppose a company issues a Rs. 21 million, three-period bond with a sinking fund obligation requiring: The issuer sink Rs. 7 million of face value after the first period and Rs. 7 million after the second.
- The issuer has an option of either buying the bonds in the market or calling them at a call price of 98.
- Assume the same interest rate tree and bond values characterizing the three-period, 9% noncallable bond described previously apply to this bond without its sinking fund agreement.
- With the sinking fund, the issuer has two options:
  - At the end of Period 1, the issuer can buy Rs. 7 million worth of the bond either at 98 or at the bond's market price.
  - At the end of Period 2, the issuer has another option to buy Rs.7 million worth of the bond either at 98 or the market price.

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Binomial Valuation of a Sinking-Fund Bond (Example)

Value of Sinking Fund
Call in Period 1
Sinking Fund CP = 98

$S_u = 11\%$

$B_u^{NSF} = 96.3612$

$IV = \text{Max}[B_u^{NSF} - CP, 0]$

$= \text{Max}[96.3612 - 98, 0] = 0$

$S_0 = 10\%,$

$$V_0^{SF(1)} = \frac{0.5(0) + 0.5(0.9335)}{(1.10)} = 0.4243$$

$S_d = 9.5\%$

$B_d^{NSF} = 98.9335$

$IV = Max[B_d^{NSF} - CP, 0]$

$= Max[98.9335 - 98, 0] = 0.9335$

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Binomial Valuation of a Sinking-Fund Bond (Example)

Value of Sinking Fund Call in Period 2
Sinking Fund CP = 98

$S_0 = 10\%,\ B_0^{NSF} = 96.9521$

$$V_0^{SF(2)} = \frac{0.5(.3095)+0.5(1.2166)}{(1.10)} = 0.6937$$

$S_u = 11\%,\ B_u^{NSF} = 96.3612$

$$V_u^{SF(2)} = \frac{0.5(0)+0.5(0.6872)}{(1.11)} = 0.3095$$

$S_d = 9.5\%,\ B_d^{NSF} = 98.9335$

$$V_d^{SF(2)} = \frac{0.5(0.6872)+0.5(1.9771)}{(1.095)} = 1.2166$$

$S_{uu} = 12.1\%,\ B_{uu}^{NSF} = 97.2346$

$IV = \text{Max}[B_{uu}^{NSF} - CP, 0]$
$= \text{Max}[97.2346 - 98, 0]$
$= 0$

$S_{ud} = 10.45\%,\ B_{ud}^{NSF} = 98.6872$

$IV = Max[B_{ud}^{NSF} - CP, 0]$
$= Max[98.6872 - 98, 0]$
$= 0.6872$

$S_{dd} = 9.025\%,\ B_{dd}^{NSF} = 99.9771$

$IV = Max[B_{dd}^{NSF} - CP, 0]$
$= Max[99.9771 - 98, 0]$
$= 1.9771$

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Binomial Valuation of a Sinking-Fund Bond (Example)

- Since each option represents 1/3 of the issue, the value of the bond's sinking fund option is 0.3727, and the value of the sinking fund bond is 96.5794 per Rs. 100 face value. Thus, the total value of the Rs. 21000000 face value issue is :

$$V_0^{SF} = (1/3)(0.4243) + (1/3)(0.6937) = 0.3727$$

$$B_0^{SF} = B_0^{NSF} - V_0^{SF}$$

$$B_0^{SF} = 96.9521 - 0.3727 = 96.5794$$

$$Issue\ Value = \frac{96.5794}{100} 21000000 = Rs.20281674$$

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CONCLUSIONS

- A putable bond gives the holder the right to sell the bond back to the issuer at a specified exercise price. From the bondholder's perspective, a put option provides a hedge against a decrease in the bond price
- The price of a putable bond will be equal to the price of an identical nonputable bond plus the value of the put option
- Sinking fund arrangement in which the issuer has the option to buy some of the bonds back either at their market price or at a call price
- The call option embedded in the sinking fund agreement makes the sinking fund valuable to the issuer

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Management of Fixed Income Securities

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Lecture 40: Binomial Valuation of Convertible Bonds

CONCEPTS COVERED

- Valuation of convertible bonds
- Valuation of callable convertible bonds

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KEYWORDS

- Conversion ratio
- Conversion value
- Straight debt value
- Conversion price

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Convertible Bonds

- A convertible bond gives the holder the right to convert the bond into a specified number of shares of stock
- To the investor, convertible bonds offer the potential for a high rate of return if the company does well and its stock price increases, while providing some downside protection as a bond if the stock declines
- Convertibles are usually callable, with the convertible bondholder usually having the right to convert the bond to stock if the issuer does call

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Convertible Bonds: Terms

- The **conversion ratio** (CR) is the number of shares of stock that can be converted when the bond is tendered for conversion.
- The **conversion value** (CV) is the convertible bond's value as a stock. At a given point in time, the conversion value is equal to the conversion ratio times the market price of the stock (P_t^S):

$$CV_t = (CR)\,P_t^S$$

- The **straight debt value** (SDV) is the convertible bond's value as a nonconvertible bond. This value is obtained by discounting the convertible's cash flows by the discount rate on a comparable non-convertible bond.
- The **conversion price** is the bond's par value divided by the conversion ratio: F/CR.

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Convertible Bonds: Minimum Price

- Arbitrage ensures that the minimum price of a convertible bond is the greater of either its straight debt value or its conversion value:

$$\text{Min } B_t^{CB} = \text{Max}[CV_t, SDV_t]$$

- If a convertible bond is priced below its conversion value, arbitrageurs could buy it, convert it to stock, and then sell the stock in the market to earn a riskless profit. Arbitrageurs seeking such opportunities would push the price of the convertible up until it is at least equal to its CV.
- If a convertible is selling below its SDV, then arbitrageurs could profit by buying the convertible and selling it as a regular bond.

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Convertible Bonds: Maximum Price

- If the convertible is callable, the call price at which the issuer can redeem the bond places a maximum limit on the convertible. That is, the issuer will find it profitable to buy back the convertible bond once its price is equal to the call price. Buying back the bond, in turn, frees the company to sell new stock or bonds at prices higher than the stock or straight debt values associated with the convertible.
- The **maximum price of a convertible is the call price.**
- The **actual price** that a convertible will trade for will be at a premium above its minimum value but below its maximum.

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Binomial Valuation of Convertible Bonds

- The valuation of a convertible bond with an embedded call is more difficult than the valuation of a bond with just one option feature.

- The valuation of convertibles needs to take into account:
  - The random patterns of interest rates
  - The random pattern of the stock's price
  - The correlation between interest rates and the stock price

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Binomial Valuation of Convertible Bonds Example

- Three-period, 10% convertible bond, Face value = 1000, Convertible into to 10 shares of the underlying company's stock (CR = 10).

Assume:

- The bond has no call option and no default risk.
- The current yield curve is flat at 5%.
- The yield curve will stay at 5% for the duration of the three periods (i.e., no market risk).
- The convertible bond's ***underlying stock price*** follows a binomial process where in each period it has an equal chance it can either increase to equal u times its initial value or decrease to equal d times the initial value,
  where u = 1.1, d = 1/1.1 = 0.9091, q = probability of stock increasing in one period = 0.5 and Current stock price is 92

- At the maturity date (end of Period 3), the bondholder will have a coupon worth 100 and will either convert the bond to stock or receive the principal of 1000.

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Binomial Valuation of Convertible Bonds Example

$Stock\ \text{Price} = P_0^S = 92,\ u = 1.10$
$d = 0.9091,\ Spot\ Rate = 5\%,$
$Convertible: Coupon = 100,\ F = 1000,$
$Conversion\ Ratio = 10$

$P_0^S = 92,\ S_0 = 5\%$
$CV = 920$
$B_0^{CB} = \dfrac{0.5(1149.32 + 100) + 0.5(1095.69 + 100)}{(1.05)}$
$= 1164.29$

$P_u^S = 101.20,\ S_u = 5\%$
$CV = 1012$
$B_u = \dfrac{0.5(1160.24 + 100) + 0.5(1053.33 + 100)}{(1.05)}$
$= 1149.64$
$B_u^{CB} = Max[CV, B_u] = 1149.32$

$P_d^S = 83.64,\ S_d = 5\%$
$CV = 836.40$
$B_d = \dfrac{0.5(1053.33 + 100) + 0.5(1047.62 + 100)}{(1.05)}$
$= 1095.69$
$B_d^{CB} = Max[CV, B_d] = 1095.69$

$P_{uu}^S = 111.32,\ S_{uu} = 5\%$
$CV = 1113.20$
$B_{uu} = \dfrac{0.5(1324.50) + 0.5(1112)}{(1.05)}$
$= 1160.24$
$B_{uu}^{CB} = Max[CV, B_{uu}] = 1160.24$

$P_{ud}^S = 92,\ S_{ud} = 5\%$
$CV = 920$
$B_{ud} = \dfrac{0.5(1112) + 0.5(1100)}{(1.05)}$
$= 1053.33$
$B_{ud}^{CB} = Max[CV, B_{ud}] = 1053.33$

$P_{dd}^S = 76.03,\ S_{dd} = 5\%$
$CV = 760.30$
$B_{dd} = \dfrac{1100}{(1.05)}$
$= 1047.62$
$B_{dd}^{CB} = \text{Max}[CV, B_{dd}] = 1047.62$

$P_{uuu}^S = 122.45$
$CV = 1224.50$
$F = 1000$
$C = 100$
$B_{uuu}^{CB} = \text{Max}[CV, F] + C$
$= 1324.50$

$P_{uud}^S = 101.20$
$CV = 1012$
$F = 1000$
$C = 100$
$B_{uud}^{CB} = \text{Max}[CV, F] + C$
$= 1112$

$P_{udd}^S = 83.64$
$CV = 836.40$
$F = 1000$
$C = 100$
$B_{udd}^{CB} = \text{Max}[CV, F] + C$
$= 1100$

$P_{ddd}^S = 69.12$
$CV = 691.20$
$F = 1000$
$C = 100$
$B_{ddd}^{CB} = \text{Max}[CV, F] + C$
$= 1100$

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Binomial Valuation of Convertible Bonds Example

In Period 3 at 122.45*:*

- The convertible bondholder would exercise her option, converting the bond to ten shares of stock. The value of the convertible bond, B^{CB}, would therefore be equal to its conversion value of 1224.50 plus the Rs. 100 coupon:

$$B_{uuu}^{CB} = \text{Max}[CV_t, F] + C$$
$$B_{uuu}^{CB} = \text{Max}[1224.50, 1000] + 100$$
$$B_{uuu}^{CB} = 1324.50$$

In Period 3 at 101.20:

- The bondholder would find it profitable to convert; thus, the value of the convertible in this case would be its conversion value of 1012 plus the Rs. 100 coupon

$$B_{uud}^{CB} = \text{Max}[CV_t, F] + C$$
$$B_{uud}^{CB} = \text{Max}[1012, 1000] + 100$$
$$B_{uud}^{CB} = 1120$$

In Period 3 at 83.64 and 69.12:

- The conversion is worthless; thus, the value of the convertible bond is equal to the principal plus the coupon: 1100.

$$B_{udd}^{CB} = \text{Max}[836.40, 1000] + 100 = 1100$$

$$B_{ddd}^{CB} = \text{Max}[691.20, 1000] + 100 = 1100$$

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Binomial Valuation of Convertible Bonds Example

In Period 2 at each node:

- The value of the convertible bond is equal to the maximum of either the present value of the bond's expected value at maturity or its conversion value.
- At all three stock prices, the present values of the bond's expected values next period are greater than the bond's conversion values.

Example: At $P^S_{uu} = 111.32$, the CV is 1113.20 compared to the convertible bond value of 1160.24; thus the value of the convertible bond is 1160.24:

$$B_{uu} = \frac{0.5[1324.50] + 0.5[1112]}{1.05} = 1160.24$$

$$B_{uu}^{CB} = Max[B_{uu}, CV] = [1160.24, 1113.20] = 1160.24$$

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Binomial Valuation of Convertible Bonds Example

In Period 1 at each node:

- The two possible bond values in Period 1 (generated by rolling the three convertible bond values in Period 2 to Period 1) also exceed their conversion values.

Current Period

- Rolling Period 1 values to the current period, we obtain a convertible bond value of ***1164.29.***

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Binomial Valuation of Callable Convertible Bonds

- With a ***callable*** convertible bond, the issuer will find it profitable to call the convertible prior to maturity whenever the price of the convertible is greater than the call price.
- When the convertible bondholder is faced with a call, she usually has the choice of either tendering the bond at the call price or converting it to stock.
- Since the issuer will call whenever the call price exceeds the convertible bond price, he is in effect forcing the holder to convert. By doing this, the issuer takes away the bondholder's value of holding the convertible, forcing the convertible bond price to equal its conversion value.

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Binomial Valuation of Callable Convertible Bonds

$Stock\ \text{Price} = P_0^S = 92,\ u = 1.10$
$d = 0.9091,\ Spot\ Rate = 5\%,$
$Convertible: Coupon = 100,\ F = 1000,$
$Conversion\ Ratio = 10,\ Callable\ at\ CP = 1100$

$P_0^S = 92,\ S_0 = 5\%$
$CV = 920$
$B_0^{CB} = \dfrac{0.5(1100+100)+0.5(1095.69+100)}{(1.05)}$
$= 1140.80$

$P_u^S = 101.20,\ S_u = 5\%$
$CV = 1012,\ CP = 1100$
$B_u = \dfrac{0.5(1113.2+100)+0.5(1053.33+100)}{(1.05)}$
$= 1126.92$
$B_u^{CB} = Max[CV, Min[B_u, CP]] = 1100$

$P_d^S = 83.64,\ S_d = 5\%$
$CV = 836.40,\ CP = 1100$
$B_d = \dfrac{0.5(1053.33+100)+0.5(1047.62+100)}{(1.05)}$
$= 1095.69$
$B_d^{CB} = Max[CV,\ Min[B_d, CP]] = 1095.69$

$P_{uu}^S = 111.32,\ S_{uu} = 5\%$
$CV = 1113.20,\ CP = 1100$
$B_{uu} = \dfrac{0.5(1324.50)+0.5(1112)}{(1.05)}$
$= 1160.24$
$B_{uu}^{CB} = Max[CV, Min[B_{uu}, CP]] = 1113.20$

$P_{ud}^S = 92,\ S_{ud} = 5\%$
$CV = 920,\ CP = 1100$
$B_{ud} = \dfrac{0.5(1112)+0.5(1100)}{(1.05)}$
$= 1053.33$
$B_{ud}^{CB} = Max[CV, Min[B_{ud}, CP]] = 1053.33$

$P_{dd}^S = 76.03,\ S_{dd} = 5\%$
$CV = 760.30,\ CP = 1100$
$B_{dd} = \dfrac{1100}{(1.05)}$
$= 1047.62$
$B_{dd}^{CB} = \text{Max}[CV, \text{Min}[B_{dd}, CP] = 1047.62$

$P_{uuu}^S = 122.45$
$CV = 1224.50$
$F = 1000$
$C = 100$
$B_{uuu}^{CB} = \text{Max}[CV, F] + C$
$= 1324.50$

$P_{uud}^S = 101.20$
$CV = 1012$
$F = 1000$
$C = 100$
$B_{uud}^{CB} = \text{Max}[CV, F] + C$
$= 1112$

$P_{udd}^S = 83.64$
$CV = 836.40$
$F = 1000$
$C = 100$
$B_{udd}^{CB} = \text{Max}[CV, F] + C$
$= 1100$

$P_{ddd}^S = 69.12$
$CV = 691.20$
$F = 1000$
$C = 100$
$B_{ddd}^{CB} = \text{Max}[CV, F] + C$
$= 1100$

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Binomial Valuation of Callable Convertible Bonds

In Period 2 at 111.32:

- The conversion value is 1113.20.
- In this case, the issuer can force the bondholder to convert by calling the bond.
- The call option therefore reduces the value of the convertible from 1160.24 to 1113.20.

In Period 2 at 92 and 76.03:

- Neither conversion by the bondholders or calling by the issuer is economical.
- Thus the bond values prevail.

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Binomial Valuation of Callable Convertible Bonds

In Period 1 at 102.20:

- The call price of 1100 is below the bond value (1126.92), but above the conversion value (1012).
- In this case, the issuer would call the bond and the holder would take the call instead of converting.
- The value of the callable convertible bond in this case would be the call price of 1100.

In Period 1 at 83.64:

- Calling and converting are not economical.
- The bond value of 1095.69 prevails.

Current Period

- Rolling Period 1's upper and lower convertible bond values to the current period, we obtain a value for the callable convertible bond of ***1140.80***.
- The callable convertible bond value is less than the noncallable convertible bond value of 1164.29.

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Binomial Valuation of Callable Convertible Bonds with Different Interest Rates (Assume uncertain spot rates)

- Use correlation or regression analysis to first estimate the relationship between a stock's price and the spot rate.
- Given a binomial model of spot rates, identify the corresponding stock prices.
- Given a binomial model of stock prices, identify the corresponding spot rates.

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Binomial Valuation of Callable Convertible Bonds with Different Interest Rates (Example)

- Suppose using regression analysis, we estimated the following relationship between the stock in our above example and the one-period spot rate:

$$S_t = 0.16 - .001P_t^S$$

- Using this equation, the corresponding spot rates associated with the stock prices from the three-period tree would be:

P^S_T	S_T
111.32	4.87%
101.20	5.90%
92.00	6.80%
83.64	7.64%
76.03	8.40%

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Binomial Valuation of Callable Convertible Bonds with Different Interest Rates

$Stock\ Price = P_0^S = 92,$
$u = 1.10,\ d = 0.9091,$
$Convertible: Coupon = 100,$
$F = 1000,\ Conversion\ Ratio = 10,$
$Callable\ at\ CP = 1100$

$P_0^S = 92,\ S_0 = 6.8\%$
$CV = 920$
$B_0^{CB} = \dfrac{0.5(1100+100)+0.5(1045.31+100)}{(1.068)}$
$= 1097.99$

$P_u^S = 101.20,\ S_u = 5.9\%$
$CV = 1012,\ CP = 1100$
$B_u = \dfrac{0.5(1113.20+100)+0.5(1035.58+100)}{(1.059)}$
$= 1108.96$
$B_u^{CB} = Max[CV, Min[B_u, CP]] = 1100$

$P_d^S = 83.64,\ S_d = 7.64\%$
$CV = 836.40,\ CP = 1100$
$B_d = \dfrac{0.5(1035.58+100)+0.5(1014.76+100)}{(1.0764)}$
$= 1045.31$
$B_d^{CB} = Max[CV,\ Min[B_d, CP]] = 1045.31$

$P_{uu}^S = 111.32,\ S_{uu} = 4.87\%$
$CV = 1113.20,\ CP = 1100$
$B_{uu} = \dfrac{0.5(1324.50)+0.5(1112)}{(1.0487)}$
$= 1161.68$
$B_{uu}^{CB} = Max[CV, Min[B_{uu}, CP]] = 1113.20$

$P_{ud}^S = 92,\ S_{ud} = 6.8\%$
$CV = 920,\ CP = 1100$
$B_{ud} = \dfrac{0.5(1112)+0.5(1100)}{(1.068)}$
$= 1035.58$
$B_{ud}^{CB} = Max[CV, Min[B_{ud}, CP]] = 1035.58$

$P_{dd}^S = 76.03,\ S_{dd} = 8.4\%$
$CV = 760.30,\ CP = 1100$
$B_{dd} = \dfrac{1100}{(1.084)}$
$= 1014.76$
$B_{dd}^{CB} = Max[CV, Min[B_{dd}, CP] = 1014.76$

$P_{uuu}^S = 122.45$
$CV = 1224.50$
$F = 1000$
$C = 100$
$B_{uuu}^{CB} = Max[CV, F] + C$
$= 1324.50$

$P_{uud}^S = 101.20$
$CV = 1012$
$F = 1000$
$C = 100$
$B_{uud}^{CB} = Max[CV, F] + C$
$= 1112$

$P_{udd}^S = 83.64$
$CV = 836.40$
$F = 1000$
$C = 100$
$B_{udd}^{CB} = Max[CV, F] + C$
$= 1100$

$P_{ddd}^S = 69.12$
$CV = 691.20$
$F = 1000$
$C = 100$
$B_{ddd}^{CB} = Max[CV, F] + C$
$= 1100$

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Binomial Valuation of Callable Convertible Bonds with Different Interest Rates (Assume uncertain spot rates)

- The exhibit shows the binomial tree of stock prices along with their corresponding spot rates given the spot rate and stock price relation.
- The convertible bond value = ***1097.99***.
- This value is lower than the previous case in which we assumed a constant yield curve at 5%.

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CONCLUSIONS

- A convertible bond gives the holder the right to convert the bond into a specified number of shares of stock
- Convertibles are usually callable, with the convertible bondholder usually having the right to convert the bond to stock if the issuer does call
- Arbitrage ensures that the minimum price of a convertible bond is the greater of either its straight debt value or its conversion value
- With a callable convertible bond, the issuer will find it profitable to call the convertible prior to maturity whenever the price of the convertible is greater than the call price

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